Eur. Phys. J. B **8**, 335–338 (1999)

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

Quantum XY spin glass model with planar Dzyaloshinskii-Moriya interactions in longitudinal field*

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Received 11 March 1998

Abstract. In the replica symmetric approximation and static limit in Matsubara "imaginary time", the quantum XY spin glass model with planar Dzyaloshinskii-Moriya interaction in longitudinal field is investigated. Several thermodynamic quantities are calculated numerically as well as spin self-interaction and spin glass order parameter for spin $S = 1/2$. It is shown that the entropy is not independent of the field. A crossover behavior of the specific heat depending on temperature is found. There is a deviation from the parabolic approximation, $C/T = A + Bh^2$.

PACS. 75.10.Nr Spin-glass and other random models – 75.50.Lk Magnetic phase boundaries (including magnetic transitions, metamagnetism, etc.) – 75.30.Gw Magnetic anisotropy – 75.40.Cx Static properties (order parameter, static susceptibility, heat capacities, critical exponents, etc.)

The properties of the glass have usually been interpreted in terms of the Sherrington-Kirkpartrick infinite-range model treated in various extensions and approximations. [1–7] Quantum spin glasses were studied for the first time by Sommers [4] and by Bray *et al.* [5] independently who treated an isotropic quantum Heisenberg spin glass model. Though the quantum problems are usually treated in "static" limit in which the noncommutativity of spin is neglected, the essential properties are revealed.

In 1983, Brodale *et al.* [9] showed that: the temperature curves of the specific heat, C, in CuMn sample, have the crossover behavior, the broad maximum in C is shifted to higher temperature with increasing applied field. The specific heat anomaly can be used to determine the transition to the spin-glass phase in CuMn. Out of de Almeida and Thouless' expectation [10], no evidence for a region of field independence of S was observed: according to the Parisi-Tolouse hypothesis, they recognized that the entropy is independent of applied field below phase boundary.

To our knowledge, there has been little attention paid to the quantum XY spin glass model with the Dzyaloshinskii-Moriya (DM) couplings between spins. This model has experimental applications due to the possibility to perform laboratory studies on spin-glass with strong anisotropy forcing the spins to align in a plane [11–16].

In the other hand, it has been found that a number of hexagonal metallic spin-glass properties are strongly influenced by various types of anisotropies [11,17,18]. A typical anisotropy is the DM couplings between spins, it may be written as

$$H_{DM} = \sum_{i<j} D_{ij}(\mathbf{S}_i \times \mathbf{S}_j). \tag{1}$$

The purpose of the present paper is to investigate the quantum infinite range XY spin glass model with infinite range random planar DM couplings in an external field for spin $S = 1/2$. The entropy and the specific heat as functions of temperature and field are calculated numerically. The corresponding spin self-interaction and the Edwards-Anderson spin glass parameters are evaluated.

The total Hamilton operator for a XY spin-glass model with DM couplings in longitudinal field reads

$$H = -\sum_{i<j} J_{ij}(S_{ix}S_{jx} + S_{iy}S_{jy}) - \sum_{i<j} D_{ij}(S_{ix}S_{jy} - S_{iy}S_{jx}) - h\sum_i S_{iy} \tag{2}$$

where the strength of exchange interactions J_{ij} and D_{ij} are quenched and distributed with Gaussian functions,

* Supported by National Natural Foundation of China under the Grant No. 19774023.
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respectively

$$P(J_{ij}) = \left(\frac{N}{2\pi J^2}\right)^{1/2} \exp\left(-\frac{NJ_{ij}^2}{2J^2}\right), \tag{3a}$$

and

$$W(D_{ij}) = \left(\frac{N}{2\pi D^2}\right)^{1/2} \exp\left(-\frac{ND_{ij}^2}{2D^2}\right), \tag{3b}$$

where J and D are defined as the mean variance of the XY exchange interaction and the DM coupling, respectively.

The deviation of the free energy is a generalization of the work by Bray and Moore [8]. The free energy function per spin reads

$$f(\mathbf{R}, \mathbf{Q}, \beta)/J = \frac{1}{4}J\beta[(1+d^2)(R_T^2 - Q_T^2) + (R_L^2 - Q_L^2) + 2d^2(R_T R_L - Q_T Q_L)] - \frac{1}{J\beta}\int Dz \ln L(z), \tag{4}$$

where $\beta = 1/k_B T$, $d = D/J$, the function $L(z)$ is defined as

$$L(z) = 2\int Dz_1 \cosh[\Omega(z, z_1)], \tag{5}$$

with

$$\Omega(z, z_1) = \frac{J\beta}{2}\sqrt{(a_1 x + a_2 x_1)^2 + (a_3 y + a_4 y_1 + \tilde{h})^2}, \tag{6}$$

$$\begin{aligned} a_1 &= \sqrt{(1+d^2)Q_T + d^2 Q_L}, \\ a_2 &= \sqrt{(1+d^2)(R_T - Q_T) + d^2(R_L - Q_L)}, \\ a_3 &= \sqrt{d^2 Q_T + Q_L}, \\ a_4 &= \sqrt{d^2(R_T - Q_T) + (R_L - Q_L)}, \\ \tilde{h} &= h/J. \end{aligned} \tag{7}$$

The abbreviation denotes

$$\langle A(z)\rangle_z = \int Dz A(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dxdy \exp\left(-\frac{x^2+y^2}{2}\right) A(z). \tag{8}$$

We calculate directly the thermodynamics functions from the free energy in equation (4) with the familiar thermodynamic formulae. The entropy of the system is given by

$$S(\mathbf{R}, \mathbf{Q}, \beta)/k_B = \left(\frac{J\beta}{2}\right)^2 [(1+d^2)(R_T^2 - Q_T^2) + (R_L^2 - Q_L^2) + 2d^2(R_T R_L - Q_T Q_L)] + \int Dz \ln L(z) - 2\int \frac{Dz}{L(z)}\int Dz_1 \Omega(z, z_1)\sinh[\Omega(z, z_1)]. \tag{9}$$

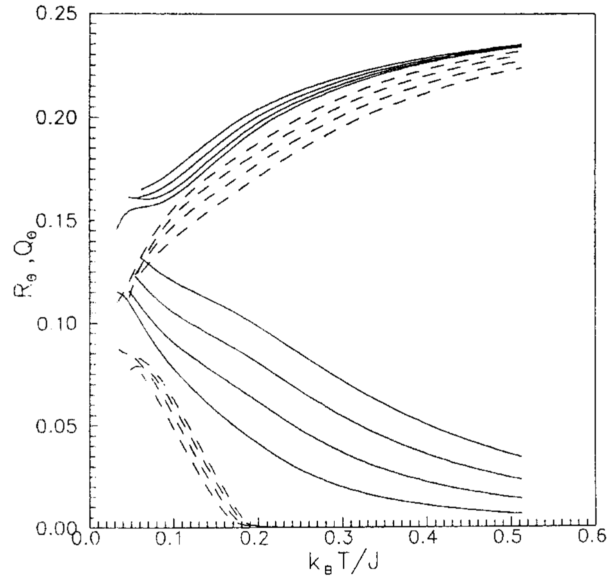

Fig. 1. The curves of the longitudinal (solid) and transverse (dashed) order parameters for different field. The up-going curves refer to R_θ's ($\theta = L, T$), the down-going curves refer to Q_θ's. From bottom to top, the curves of R_L and Q_L correspond to $\tilde{h} = h/J = 0.16$, 0.24, 0.32, and 0.40. From top to bottom, the curves of R_T and Q_T correspond to $\tilde{h} = 0.16$, 0.24, 0.32, and 0.40, where $d = 0.2$, $S = 1/2$.

The specific heat can be calculated from the formula

$$C = -\beta\frac{dS(\mathbf{R}, \mathbf{Q}, \beta)}{d\beta}\cdot \tag{10}$$

We do not give the final result of specific heat because of its complication. The function $f(\mathbf{Q}, \mathbf{R}, \beta)$ in equation (4) must be evaluated at saddle point with respect to the spin self-interactions and the spin-glass order parameters.

This condition gives the following self-consistency equations

$$R_T = \frac{2}{(a_2 J\beta)^2}\int \frac{Dz}{L(z)}\int Dz_1 \cosh[\Omega(z, z_1)](x_1^2 - 1), \tag{11}$$

$$R_L = \frac{2}{(a_4 J\beta)^2}\int \frac{Dz}{L(z)}\int Dz_1 \cosh[\Omega(z, z_1)](y_1^2 - 1), \tag{12}$$

$$Q_T = \frac{2}{(a_2 J\beta)^2}\int Dz\left[\frac{1}{L(z)}\int Dz_1 \cosh[\Omega(z, z_1)x_1\right]^2, \tag{13}$$

$$Q_L = \frac{2}{(a_4 J\beta)^2}\int Dz\left[\frac{1}{L(z)}\int Dz_1 \cosh[\Omega(z, z_1)]y_1\right]^2. \tag{14}$$

Figure 1 shows the dependence of the longitudinal and the transverse components of spin self-interactions and spin glass order parameters on temperature for different fields with spin $S = 1/2$. It is clear to see that the longitudinal components Q_L and R_L of order parameters increase

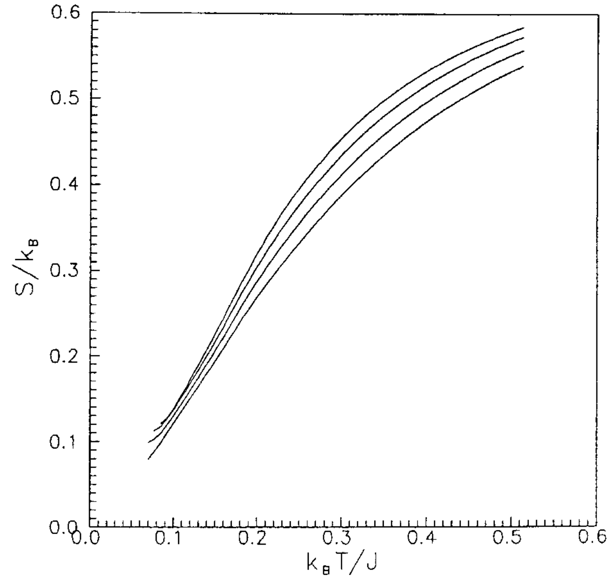

Fig. 2. The curves of the entropy depending on the reduced temperature k_BT/J for different field. From top to bottom, the curves of S correspond to $\tilde{h} = 0.16$, 0.24, 0.32, and 0.40. Where $d = 0.2$, $S = 1/2$.

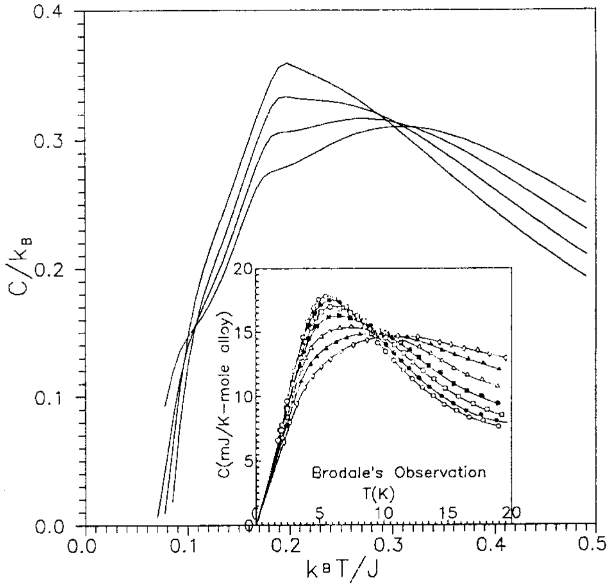

Fig. 4. The curves of the specific heat depending on the temperature for different field. In the right side, from bottom to top, the curves of C correspond to $\tilde{h} = 0.16$, 0.24, 0.32, and 0.40, where $d = 0.2$, $S = 1/2$.

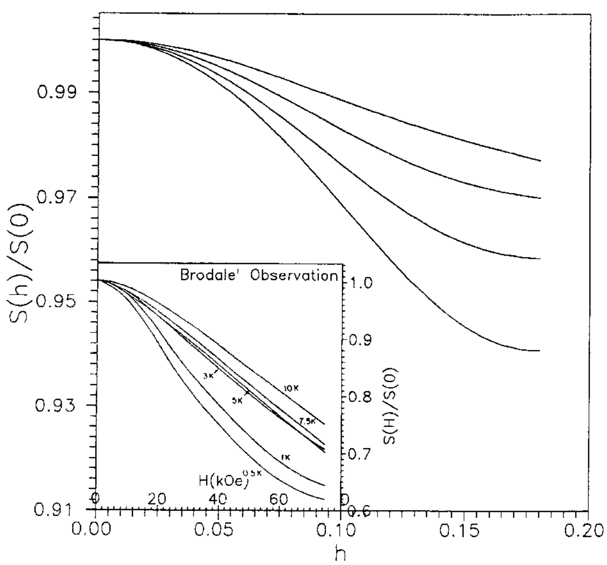

Fig. 3. The entropy as function of $\tilde{h}$. From bottom to top, the reduced temperature k_BT/J is 0.7575, 0.7625, 0.7675 and 0.7725 respectively, where $d = 0.2$, $S = 1/2$. The insert shows Brodale's experimental result.

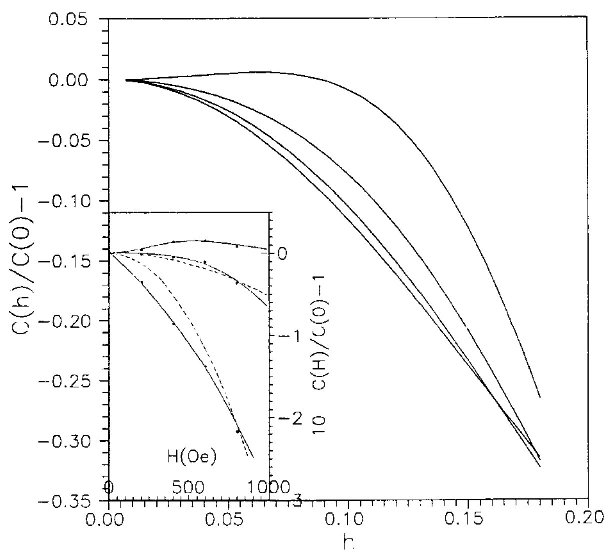

Fig. 5. The parabolic approximation, $C/T = A + B\tilde{h}^2$. From top to bottom, the curves correspond to $k_BT/J = 0.7575$, 0.7625, 0.7675 and 0.7725, where $d = 0.2$, $S = 1/2$. The insert shows Brodale's experimental result.

with the increasing field while the transverse components Q_T and R_T decreases with the increasing field. At the transition point, the transverse spin-glass parameter Q_T becomes zero.

The dependence of the entropy on temperature for spin $S = 1/2$ is plotted in Figure 2 for different fields. The entropy is positive in the temperature region considered.

The field dependence of the entropy, $S(\tilde{h})/S(0)$, for different temperature is illustrated in Figure 3. There is no feature in these curves that obviously corresponds to the hypothesis [10] that S is independent of h in the spin glass phase, agreeing with the experimental observation [9].

The temperature-specific heat curves are shown in Figure 4 for different fields. The curves of the specific heat *vs.* temperature for different fields have a crossover behavior, and are shifted to progressively higher temperature with increasing field. Compared with Figure 1, we can see that there are anomalies in the curves which correspond to the transition points (the boundaries of the Q_T's $\neq 0$ and Q_T's $= 0$).

The weak field dependence of the specific heat $C(\tilde{h})/C(0) - 1$ is illustrated in Figure 5. One can see that there are deviations from the parabolic approximation, $C/T = A + Bh^2$, where h denotes an applied field. This result agrees with the experimental results very well [9].

In summary, the quantum XY spin glass model with the planar DM interaction in external field for spins $S = 1/2$ is investigated theoretically. Numerical calculations show that the entropy is positive [7]. The entropy is not independent of the field. The specific heat depending on the temperature has crossover behavior. There is deviation from the parabolic approximation, $C/T = A + Bh^2$.

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